Calcul des déformations des fils élastiques

Fils élastiques en arc de cercle - Accélération parallèle à l'axe du fil

Force gravitationnelle agissant perpendiculairement au plan du fil

Fil rond en cuivre

$d := 0.23 \cdot mm$ $\quad S := \pi \cdot \frac{d^2}{4}$ $\quad E := 1.1 \cdot 10^5 \cdot N \cdot mm^{-2}$ $\quad G := \frac{E}{2.6}$ $\quad \rho := 8.9 \cdot 10^3 \cdot kg \cdot m^{-3}$

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$J_t := J_{t\_circ}(d)$ $\quad I_{22} := I_{f\_circ}(d)$ $\quad I_{33} := I_{22}$

$W_t := W_{t\_circ}(d)$ $\quad W'_t := W_t$ $\quad W_{f2} := W_{f\_circ}(d)$ $\quad W_{f3} := W_{f2}$

Caractéristiques de l'arc de cercle $\quad R := 21.5 \cdot mm$ $\quad \psi_{AB} := 75 \cdot deg$

Forces extérieures en bout d'arc $\quad \psi_F := \psi_{AB}$ $\quad \psi_q := \psi_{AB}$

$F_x := 0 \cdot N$ $\quad F_y := 0 \cdot N$ $\quad F_z := 0 \cdot N$ $\quad C_x := 0 \cdot N \cdot mm$ $\quad C_y := 0 \cdot N \cdot mm$ $\quad C_z := 0 \cdot N \cdot mm$

Force distribuée gravitationnelle $\quad q_0 := \rho \cdot g \cdot S$ $\quad P_{fil} := q_0 \cdot R \cdot \psi_{AB}$ $\quad P_{fil} = 1.021 \times 10^{-4} N$

$q_x(\chi) := 0 \cdot N \cdot m^{-1}$ $\quad q_y(\chi) := 0 \cdot N \cdot m^{-1}$ $\quad q_z(\chi) := -q_0$

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Arc de cercle E_L - F&C&q.mcd(R)

Valeur de tests transitoires $\quad \alpha_m := 20 \cdot deg$

Torseur des forces de cohésion

$\mathbf{M_{cq}}(\psi_F, \psi_q, \alpha_m)^T = \begin{pmatrix} -5.91 \times 10^{-4} & -4.662 \times 10^{-4} & 0 \end{pmatrix} N \cdot mm$

Sollicitations

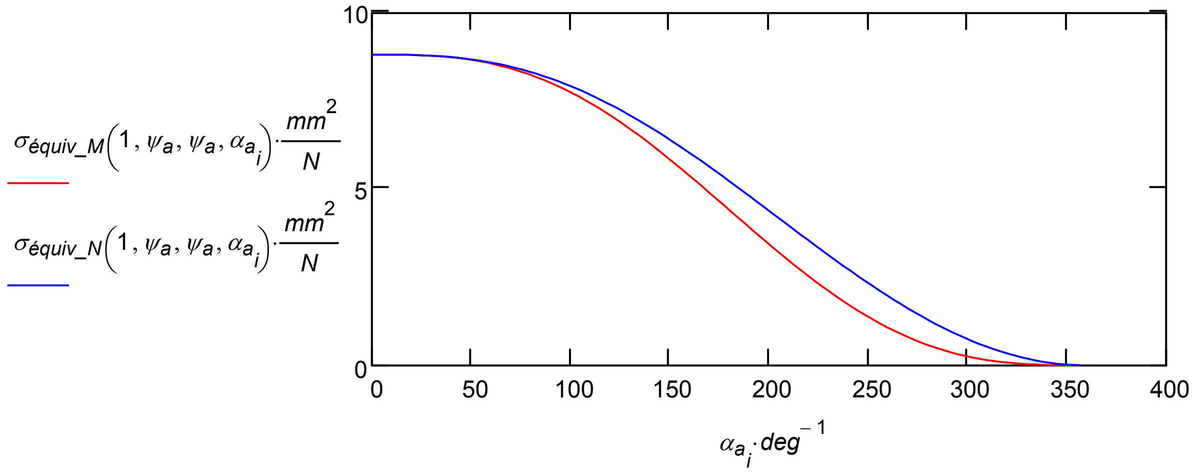
$\mathbf{e'_1}(\alpha_m)^T = (-0.342 \quad 0.94 \quad 0)$ $\quad \mathbf{e'_2}(\alpha_m)^T = (-0.94 \quad -0.342 \quad 0)$ $\quad \mathbf{e'_3}(\alpha_m)^T = (0 \quad 0 \quad 1)$

Moment de torsion $\quad M_t(\psi_F, \psi_q, \alpha_m) = -2.36 \times 10^{-4} N \cdot mm$

Moments de flexion $\quad M_{f2}(\psi_F, \psi_q, \alpha_m) = 7.148 \times 10^{-4} N \cdot mm$ $\quad M_{f3}(\psi_F, \psi_q, \alpha_m) = 0 \, N \cdot mm$

Contraintes

Cas d'un anneau fendu $\quad n := 101$ $\quad i := 1 .. n-1$ $\quad \psi_a := 360 \cdot deg$ $\quad \alpha_{a_i} := (i-1) \cdot \frac{\psi_a}{n-1}$



Calcul des déplacements par les intégrales de Mohr

Position du déplacement désiré $\alpha_M := 40 \cdot deg$

Calcul des déplacements linéiques

Déplacement dans la direction de Ox $\lambda := 0 \cdot deg$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda,\gamma)| = 1$

$\delta_{tv}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv2}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv3}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$

$\delta_x(\alpha) := \delta_v(\psi_F, \psi_q, \alpha, \lambda, \gamma)$ $\boxed{\delta_x(\alpha_M) = 0\,mm}$

Déplacement dans la direction de Oy $\lambda := 90 \cdot deg$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda,\gamma)| = 1$

$\delta_{tv}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv2}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv3}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$

$\delta_y(\alpha) := \delta_v(\psi_F, \psi_q, \alpha, \lambda, \gamma)$ $\boxed{\delta_y(\alpha_M) = 0\,mm}$

Déplacement dans la direction de Oz $\lambda := 0 \cdot deg$ $\gamma := 0 \cdot deg$ $|\mathbf{v}(\lambda,\gamma)| = 1$

$\delta_{tv}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = -8.69 \times 10^{-4}\,mm$ $\delta_{fv2}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = -6.34 \times 10^{-3}\,mm$

$\delta_{fv3}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_z(\alpha) := \delta_v(\psi_F, \psi_q, \alpha, \lambda, \gamma)$ $\boxed{\delta_z(\alpha_M) = -7.213 \times 10^{-3}\,mm}$

Calcul des déplacements angulaires

Déplacement angulaire autour de Ox $\lambda_c := 0 \cdot deg$ $\gamma_c := 90 \cdot deg$ $|\mathbf{cv}(\lambda,\gamma)| = 1$

$\theta_{tcv}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = 4.5 \times 10^{-3}\,deg$ $\theta_{fcv2}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = -0.04\,deg$

$\theta_{fcv3}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_x(\alpha) := \theta_{cv}(\psi_F, \psi_q, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_x(\alpha_M) = -0.035\,deg}$

Déplacement angulaire autour de Oy $\lambda_c := 90 \cdot deg$ $\gamma_c := 90 \cdot deg$ $|\mathbf{cv}(\lambda,\gamma)| = 1$

$\theta_{tcv}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = -0.02\,deg$ $\theta_{fcv2}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = -0.01\,deg$

$\theta_{fcv3}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_y(\alpha) := \theta_{cv}(\psi_F, \psi_q, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_y(\alpha_M) = -0.03\,deg}$

Déplacement angulaire autour de Oz $\lambda_c := 0 \cdot deg$ $\gamma_c := 0 \cdot deg$ $|\mathbf{cv}(\lambda,\gamma)| = 1$

$\theta_{tcv}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_{fcv2}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$

$\theta_{fcv3}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_z(\alpha) := \theta_{cv}(\psi_F, \psi_q, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_z(\alpha_M) = 0\,deg}$

Déplacement angulaire de flexion $\lambda_c := \alpha_M$ $\gamma_c := 90 \cdot deg$ $|\mathbf{cv}(\lambda,\gamma)| = 1$

$\theta_{tcv}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = -8.55 \times 10^{-3}\,deg$ $\theta_{fcv2}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = -0.04\,deg$

$\theta_{fcv3}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_f(\alpha) := \theta_{cv}(\psi_F, \psi_q, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_f(\alpha_M) = -0.046\,deg}$

Déplacement angulaire de torsion $\lambda_c := \alpha_M + \frac{\pi}{2}$ $\gamma_c := 90 \cdot deg$ $|\mathbf{cv}(\lambda,\gamma)| = 1$

$\theta_{tcv}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = -0.02\,deg$ $\theta_{fcv2}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = 0.017\,deg$

$\theta_{fcv3}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_t(\alpha) := \theta_{cv}(\psi_F, \psi_q, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_t(\alpha_M) = -2.872 \times 10^{-4}\,deg}$

Solution analytique

Moment de torsion $\quad M_t(\psi,\alpha') := q_0 \cdot R^2 \cdot [\sin(\psi-\alpha') - (\psi-\alpha')]$

Moment fléchissant $\quad M_{f2}(\psi,\alpha') := q_0 \cdot R^2 \cdot (1-\cos(\psi-\alpha'))$

Déplacement selon Oz $\quad M_{tv}(\alpha,\alpha') := R\cdot(1-\cos(\alpha-\alpha')) \qquad M_{fv}(\alpha,\alpha') := -R\cdot\sin(\alpha-\alpha')$

$$\delta_{qt3}(\psi,\alpha) := \frac{R}{G\cdot J_t}\cdot\int_0^{\alpha} M_t(\psi,\alpha')\cdot M_{tv}(\alpha,\alpha')\,d\alpha' \qquad \delta_{qf3}(\psi,\alpha) := \frac{R}{E\cdot I_{22}}\cdot\int_0^{\alpha} M_{f2}(\psi,\alpha')\cdot M_{fv}(\alpha,\alpha')\,d\alpha'$$

$$I_t(\psi,\alpha) := \sin(\alpha)\cdot\sin(\psi) - (\sin(\psi-\alpha) + 2\cdot\psi - \alpha)\cdot\alpha + 2\cdot[\cos(\alpha) + \sin(\alpha)\cdot\psi - \cos(\psi)\cdot(1-\cos(\alpha)) - 1]$$

$$I_f(\psi,\alpha) := \sin(\alpha)\cdot\sin(\psi) - \alpha\cdot\sin(\psi-\alpha) + 2\cdot(\cos(\alpha)-1)$$

$$\delta_{qt3}(\psi,\alpha) := \frac{q_0\cdot R^4}{2\cdot G\cdot J_t}\cdot I_t(\psi,\alpha) \qquad \delta_{qf3}(\psi,\alpha) := \frac{q_0\cdot R^4}{2\cdot E\cdot I_{22}}\cdot I_f(\psi,\alpha) \qquad \delta_{q3}(\psi,\alpha) := \delta_{qt3}(\psi,\alpha) + \delta_{qf3}(\psi,\alpha)$$

$$\delta_{qt3}(\psi_F,\alpha_M) = -8.688\times 10^{-4}\,mm \qquad \delta_{qf3}(\psi_F,\alpha_M) = -6.345\times 10^{-3}\,mm \qquad \delta_{q3}(\psi_F,\alpha_M) = -7.213\times 10^{-3}\,mm$$

Déplacement angulaire de flexion $\quad M_{tv}(\alpha,\alpha') := \sin(\alpha-\alpha') \qquad M_{fv}(\alpha,\alpha') := -\cos(\alpha-\alpha')$

$$\theta_{ft}(\psi,\alpha) := \frac{R}{G\cdot J_t}\cdot\int_0^{\alpha} M_t(\psi,\alpha')\cdot M_{tv}(\alpha,\alpha')\,d\alpha' \qquad \theta_{ff}(\psi,\alpha) := \frac{R}{E\cdot I_{22}}\cdot\int_0^{\alpha} M_{f2}(\psi,\alpha')\cdot M_{fv}(\alpha,\alpha')\,d\alpha'$$

$$\theta_{ft}(\psi,\alpha) := \frac{q_0\cdot R^3}{2\cdot G\cdot J_t}\cdot[\alpha\cdot\cos(\psi-\alpha) + 2\cdot\alpha - \sin(\alpha)\cdot(\cos(\psi)+2) - 2\cdot\psi\cdot(1-\cos(\alpha))]$$

$$\theta_{ff}(\psi,\alpha) := \frac{q_0\cdot R^3}{2\cdot E\cdot I_{22}}\cdot[\alpha\cdot\cos(\psi-\alpha) + \sin(\alpha)\cdot(\cos(\psi)-2)] \qquad \theta_f(\psi,\alpha) := \theta_{ft}(\psi,\alpha) + \theta_{ff}(\psi,\alpha)$$

$$\theta_{ft}(\psi_F,\alpha_M) = -8.553\times 10^{-3}\,deg \qquad \theta_{ff}(\psi_F,\alpha_M) = -0.037\,deg \qquad \theta_f(\psi_F,\alpha_M) = -0.046\,deg$$

Déplacement angulaire de torsion $\quad M_{tv}(\alpha,\alpha') := \cos(\alpha-\alpha') \qquad M_{fv}(\alpha,\alpha') := \sin(\alpha-\alpha')$

$$\theta_{tt}(\psi,\alpha) := \frac{R}{G\cdot J_t}\cdot\int_0^{\alpha} M_t(\psi,\alpha')\cdot M_{tv}(\alpha,\alpha')\,d\alpha' \qquad \theta_{tf}(\psi,\alpha) := \frac{R}{E\cdot I_{22}}\cdot\int_0^{\alpha} M_{f2}(\psi,\alpha')\cdot M_{fv}(\alpha,\alpha')\,d\alpha'$$

$$\theta_{tt}(\psi,\alpha) := \frac{q_0\cdot R^3}{2\cdot G\cdot J_t}\cdot[\alpha\cdot\sin(\psi-\alpha) + \sin(\alpha)\cdot\sin(\psi) - 2\cdot\psi\cdot\sin(\alpha) + 2\cdot(1-\cos(\alpha))]$$

$$\theta_{tf}(\psi,\alpha) := \frac{q_0\cdot R^3}{2\cdot E\cdot I_{22}}\cdot[\alpha\cdot\sin(\psi-\alpha) - \sin(\alpha)\cdot\sin(\psi) + 2\cdot(1-\cos(\alpha))] \qquad \theta_t(\psi,\alpha) := \theta_{tt}(\psi,\alpha) + \theta_{tf}(\psi,\alpha)$$

$$\theta_{tt}(\psi_F,\alpha_M) = -0.017\,deg \qquad \theta_{tf}(\psi_F,\alpha_M) = 0.017\,deg \qquad \theta_t(\psi_F,\alpha_M) = -2.872\times 10^{-4}\,deg$$

Cas particuliers

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Définition Atan.mcd(R)

Quart de cercle

$\psi := 90\cdot deg \qquad L := R\cdot\psi \qquad L = 33.772\,mm$

$\delta_{q3}(\psi,\psi) = -0.036\,mm \qquad \theta_t(\psi,\psi) = 0.056\,deg \qquad \theta_f(\psi,\psi) = -0.067\,deg$

$$\Delta_{90} := \frac{q_0\cdot R^4}{2}\cdot\left[\frac{0}{N\cdot m^2} \quad \frac{0}{N\cdot m^2} \quad \left[\frac{-1}{E\cdot I_{22}} + \left(-1+\pi-\frac{\pi^2}{4}\right)\cdot\frac{1}{G\cdot J_t}\right]\right]^T \qquad \Delta_{90} = \begin{pmatrix}0\\0\\-0.036\end{pmatrix}mm$$

$$\Theta_t := \frac{q_0\cdot R^3}{2}\cdot\left(\frac{1}{E\cdot I_{22}} + \frac{3-\pi}{G\cdot J_t}\right) \qquad \Theta_t = 0.056\,deg \qquad \Theta_f := \frac{q_0\cdot R^3}{4}\cdot\left(\frac{\pi-4}{E\cdot I_{22}} + \frac{\pi-4}{G\cdot J_t}\right) \qquad \Theta_f = -0.067\,deg$$

Graphe de la déformation

$x_0(\alpha) := R\cdot\cos(\alpha) \qquad y_0(\alpha) := R\cdot\sin(\alpha) \qquad z_0(\alpha) := 0$

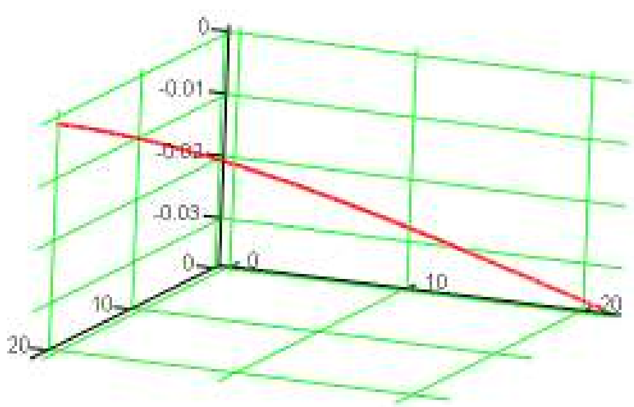
$$z_d(\alpha) := \delta_{q3}(\psi,\alpha) \qquad \theta_d(\alpha) := \arctan\left(\frac{z_d(\alpha)}{R}\right) \qquad x_d(\alpha) := R\cdot\cos(\theta_d(\alpha))\cdot\cos(\alpha) \qquad y_d(\alpha) := R\cdot\cos(\theta_d(\alpha))\cdot\sin(\alpha)$$

$$x'_d(\alpha) := \frac{d}{d\alpha}x_d(\alpha) \qquad y'_d(\alpha) := \frac{d}{d\alpha}y_d(\alpha) \qquad z'_d(\alpha) := \frac{d}{d\alpha}z_d(\alpha)$$

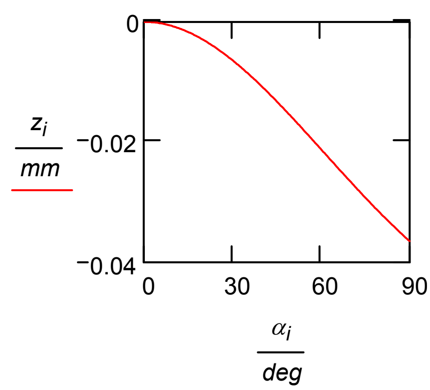
$L = 33.772\,mm$

$$\alpha_0 := 0 \qquad \alpha_{max} := \psi \qquad L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2 + z'_d(\alpha)^2}\,d\alpha \qquad L_d = 33.772\,mm$$

$$n := 201 \qquad i := 1..n \qquad \alpha_i := \frac{\psi}{n-1}\cdot(i-1) \qquad z_i := z_d(\alpha_i) \qquad x_i := x_d(\alpha_i) \qquad y_i := y_d(\alpha_i)$$



$\left(\frac{x}{mm}, \frac{y}{mm}, \frac{z}{mm}\right)$



Demi-cercle

$\psi := 180 \cdot deg \qquad L := R \cdot \psi \qquad L = 67.544\,mm$

$\delta_{q3}(\psi,\psi) = -0.432\,mm \qquad \theta_t(\psi,\psi) = 0.629\,deg \qquad \theta_f(\psi,\psi) = -0.064\,deg$

$$\Delta_{180} := \frac{q_0 \cdot R^4}{2} \cdot \left[\frac{0}{N \cdot m^2} \quad \frac{0}{N \cdot m^2} \quad \left(\frac{-4}{E \cdot I_{22}} + \frac{-\pi^2}{G \cdot J_t} \right) \right]^T \qquad \Delta_{180} = \begin{pmatrix} 0 \\ 0 \\ -0.432 \end{pmatrix} mm$$

$$\Theta_t := \frac{q_0 \cdot R^3}{2} \cdot \left(\frac{4}{E \cdot I_{22}} + \frac{4}{G \cdot J_t} \right) \qquad \Theta_t = 0.629\,deg \qquad \Theta_f := \frac{q_0 \cdot R^3}{2} \cdot \left(\frac{\pi}{E \cdot I_{22}} + \frac{-\pi}{G \cdot J_t} \right) \qquad \Theta_f = -0.064\,deg$$

Graphe de la déformation

$x_0(\alpha) := R \cdot cos(\alpha) \qquad y_0(\alpha) := R \cdot sin(\alpha) \qquad z_0(\alpha) := 0$

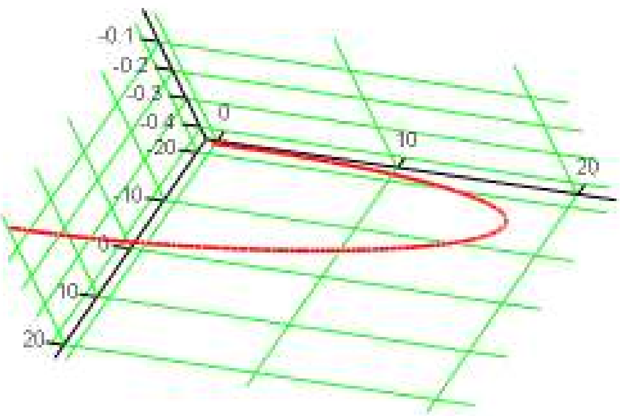
$$z_d(\alpha) := \delta_{q3}(\psi,\alpha) \qquad \theta_d(\alpha) := arctan\left(\frac{z_d(\alpha)}{R} \right) \qquad x_d(\alpha) := R \cdot cos(\theta_d(\alpha)) \cdot cos(\alpha) \qquad y_d(\alpha) := R \cdot cos(\theta_d(\alpha)) \cdot sin(\alpha)$$

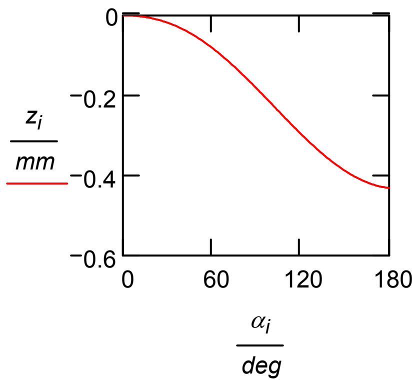
$$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha) \qquad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha) \qquad z'_d(\alpha) := \frac{d}{d\alpha} z_d(\alpha)$$

$L = 67.544\,mm$

$$\alpha_0 := 0 \qquad \alpha_{max} := \psi \qquad L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2 + z'_d(\alpha)^2}\, d\alpha \qquad L_d = 67.541\,mm$$

$$n := 201 \qquad i := 1..n \qquad \alpha_i := \frac{\psi}{n-1} \cdot (i-1) \qquad z_i := z_d(\alpha_i) \qquad x_i := x_d(\alpha_i) \qquad y_i := y_d(\alpha_i)$$





$$\left(\frac{x}{mm}, \frac{y}{mm}, \frac{z}{mm} \right)$$

Anneau fendu

$\psi := 360 \cdot deg \qquad L := R \cdot \psi \qquad L = 135.088\,mm$

$\delta_{q3}(\psi,\psi) = -1.316\,mm \qquad \theta_t(\psi,\psi) = 0\,deg \qquad \theta_f(\psi,\psi) = 2.104\,deg$

$$\Delta_{360} := \frac{q_0 \cdot R^4}{2} \cdot \left[\frac{0}{N \cdot m^2} \quad \frac{0}{N \cdot m^2} \quad \left(\frac{0}{E \cdot I_{22}} + \frac{-4 \cdot \pi^2}{G \cdot J_t} \right) \right]^T \qquad \Delta_{360} = \begin{pmatrix} 0 \\ 0 \\ -1.316 \end{pmatrix} mm$$

$$\Theta_t := \frac{q_0 \cdot R^3}{2} \cdot \left(\frac{0}{E \cdot I_{22}} + \frac{0}{G \cdot J_t} \right) \qquad \Theta_t = 0\,deg \qquad \Theta_f := \frac{q_0 \cdot R^3}{2} \cdot \left(\frac{2 \cdot \pi}{E \cdot I_{22}} + \frac{6 \cdot \pi}{G \cdot J_t} \right) \qquad \Theta_f = 2.104\,deg$$

Graphe de la déformation

$x_0(\alpha) := R \cdot cos(\alpha) \qquad y_0(\alpha) := R \cdot sin(\alpha) \qquad z_0(\alpha) := 0$

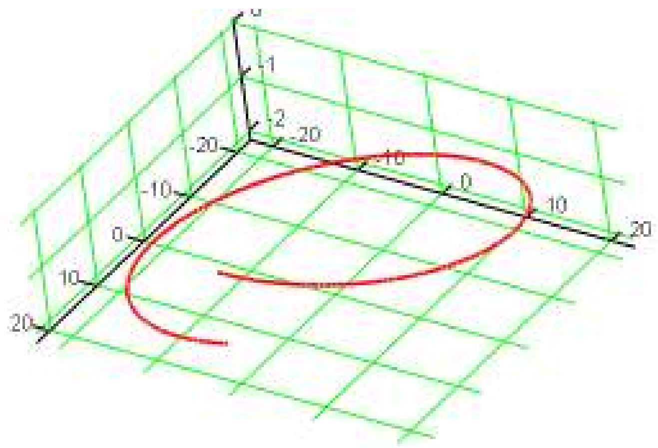
$$z_d(\alpha) := \delta_{q3}(\psi,\alpha) \qquad \theta_d(\alpha) := arctan\left(\frac{z_d(\alpha)}{R}\right) \qquad x_d(\alpha) := R \cdot cos(\theta_d(\alpha)) \cdot cos(\alpha) \qquad y_d(\alpha) := R \cdot cos(\theta_d(\alpha)) \cdot sin(\alpha)$$

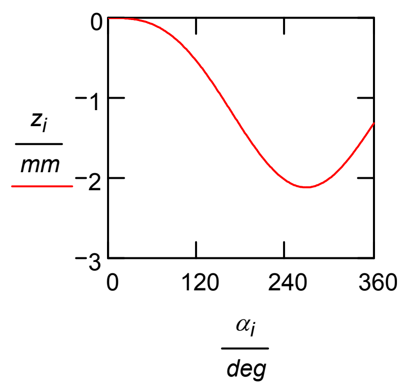
$$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha) \qquad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha) \qquad z'_d(\alpha) := \frac{d}{d\alpha} z_d(\alpha)$$

$L = 135.088\,mm$

$$\alpha_0 := 0 \qquad \alpha_{max} := \psi \qquad L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2 + z'_d(\alpha)^2}\, d\alpha \qquad L_d = 134.858\,mm$$

$$n := 201 \qquad i := 1..n \qquad \alpha_i := \frac{\psi}{n-1} \cdot (i-1) \qquad z_i := z_d(\alpha_i) \qquad x_i := x_d(\alpha_i) \qquad y_i := y_d(\alpha_i)$$





$$\left(\frac{x}{mm}, \frac{y}{mm}, \frac{z}{mm} \right)$$